A Walrasian Mechanism with Markups for Nonconvex Markets

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Overview

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Price-based mechanism have key advantages: they economize on communication, and provide helpful incentives (own effect=price effect).

Wanted: a price-based mechanism for **nonconvex economies** with **many products** that is computable, resource-feasible, and nearly efficient, with no budget deficits.

Our approach

We introduce **markup equilibrium**: a triple $(\mathbf{x}, p, \alpha)$ consisting of a resource-feasible allocation $\mathbf{x}$, a price vector p, and a markup parameter $\alpha \geq 0$ such that:

(a) payments and allocations for sellers are guided by price vector p,

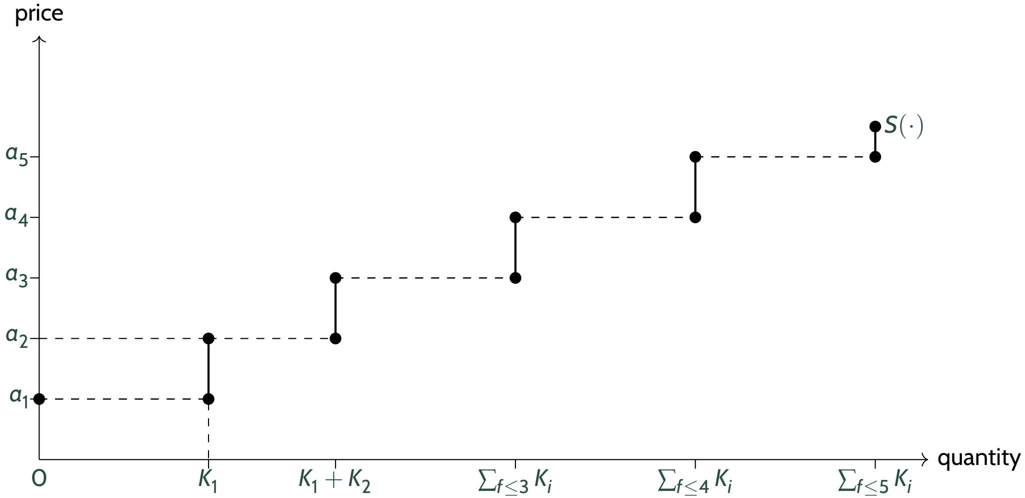
(b) payments and allocations for buyers are guided by price vector $(1 + \alpha)p$; and

(c) payments from buyers exceed payments to sellers.

A **markup mechanism** is a mechanism that outputs a markup equilibrium.

Example

Firms can produce exactly K_i units at cost a_iK_i.



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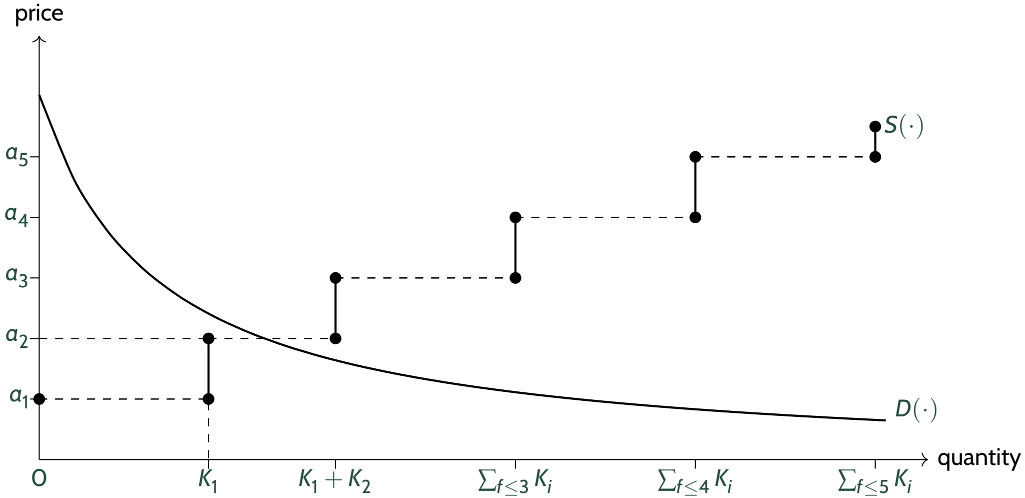

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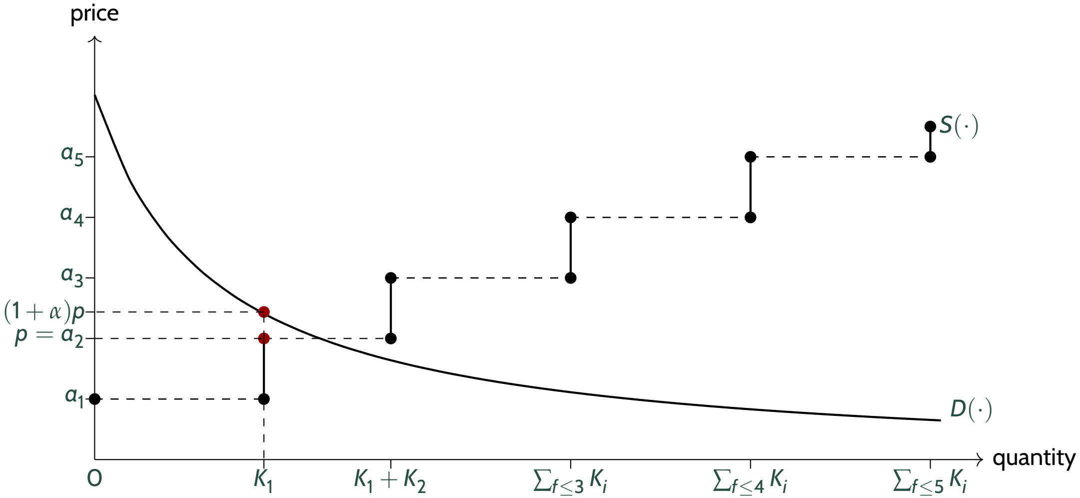

Main results

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If moreover, the number of buyers and sellers grows large ($N \to \infty$) and the limiting economy is well-behaved (in a technical sense), there is a markup mechanism with $O(1/N)$ markup, leading to $O(1)$ deadweight loss $\rightsquigarrow$ **approximate efficiency**.

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Additional Tools: a new welfare bound—the **Bound-Form First Welfare Theorem**—that links welfare loss to rationing and excess supply, and stronger **ex post incentive results** for the Walrasian mechanism (see also Watt, 2025).

Related literature

- **Nonconvexity and Approximate Equilibrium.** Farrell (1959), Rothenberg (1960), Koopmans (1961), Bator (1961), Starr (1969), Heller (1972), Nguyen & Vohra (2024).
  - Classical approximate equilibrium notions relax resource / budget feasibility.
  - **This paper:** keeps **exact feasibility**, and relaxes **efficiency** of a mechanism.

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- **Competitive Equilibrium with Indivisibilities / Substitutes.** Henry (1970), Kelso & Crawford (1982), Bikhchandani & Mamer (1997), Gul & Stacchetti (1999), Danilov et al. (2001), Sun & Yang (2006), Milgrom & Strulovici (2009), Hatfield et al. (2013), Baldwin & Klemperer (2019), Baldwin et al. (2023), Nguyen & Vohra (2024).
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- **Incentives in Large Markets.** Roberts & Postlewaite (1976), Jackson (1992), Azevedo & Budish (2019), Watt (2025).
  - **This paper:** extends logic to **markup mechanisms** in nonconvex economies.

Model

Agents and preferences

Consumers $n \in N$

- consumption bundles $x_n \in$ compact $X_n \subset \mathbb{R}^L_+$
- **quasilinear** preferences $U_n(x_n, t) = u_n(x_n) - t$
- **indirect utility** at $p \in \mathbb{R}^L_+$ is $\hat{u}_n(p) = \max_{x_n \in X_n} \{u_n(x_n) - p \cdot x_n\}$, maximized at **demand set** $D_n(p)$.

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Allocation $\mathbf{x} = ((x_n)_{n \in N}, (y_f)_{f \in F})$ is **feasible** if $x_n \in X_n$ for all n, $y_f \in Y_f$ for all f, and $\sum_n x_n \leq \sum_f y_f$.

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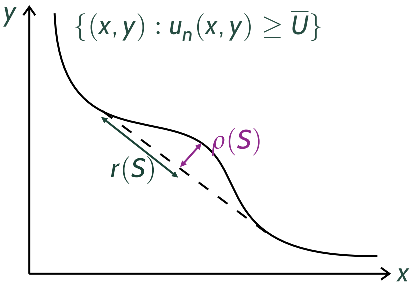
Surplus of $\mathbf{x} \in \mathbf{X}$ is $\mathcal{S}(\mathbf{x}) = \sum_{n \in N} u_n(x_n) - \sum_{f \in F} c_f(y_f)$, maximized by **efficient** allocation x^*.

Convex vs nonconvex economies

Measuring **nonconvexity** for buyers (analogous for sellers):

- upper contour sets $UC_n^{\bar{u}} = \{x : u_n(x) \in X_n \geq \bar{u}\}$
- inner radius $r(S) = \sup_{x \in \mathrm{co}(S)} \inf_{T \subseteq S : x \in \mathrm{co}(T)} \mathrm{rad}(T)$
- inner distance $\rho(S) = \sup_{x \in \mathrm{co}(S)} \inf_{y \in S} \|x - y\|$
- preference nonconvexity: $\rho_n = \max_{\bar{u} \in \mathbb{R}} \rho(UC_n^{\bar{u}})$
- market nonconvexity: $\rho_{\mathcal{E}} = \max\{\max_n \rho_n, \max_F \rho_F\}$.



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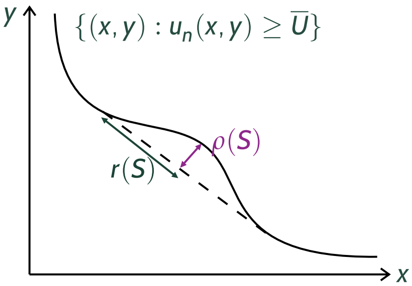

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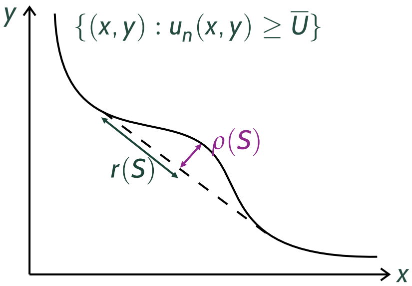

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Nonconvexity $\rightsquigarrow$ possibility of **nonexistence** $\rightsquigarrow$ any $(\mathbf{x}, p)$ with $x_n \in D_n(p)$ and $y_f \in S_f(p)$ exhibits:

- supply shortfall $\sum_n x_n^\ell > \sum_f y_f^\ell$ and/or
- budget deficit $\sum_n x_n^\ell < \sum_f y_f^\ell$.

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Fix a (feasible) allocation-price pair $(\mathbf{x}, p)$.

Define consumer's **rationing loss** as $\mathcal{R}_n(x_n, p) = \hat{u}_n(p) - (u_n(x_n) - p \cdot x_n)$.

Define producer's **rationing loss** as $\mathcal{R}_f(y_f, p) = \hat{\pi}_f(p) - (p \cdot y_f - c_f(y_f))$.

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Bound-Form First Welfare Theorem. Given any price p, the deadweight loss of allocation $\mathbf{x} \in \Omega$ satisfies

$$\mathcal{S}^* - \mathcal{S}(\mathbf{x}) \leq \underbrace{p \cdot \left(\sum_f y_f - \sum_n x_n \right)}_{\text{budget deficit}} + \underbrace{\sum_n \mathcal{R}_n(x_n, p) + \sum_f \mathcal{R}_f(y_f, p)}_{\text{rationing losses}} .$$

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Corollary: if $(\mathbf{x}, p)$ is a Walrasian equilibrium, FWT obtains.

Proof

Let $\mathbf{x}^*$ be the efficient allocation. By definition

$$\hat{u}_n(p) \geq u_n(x_n^*) - p \cdot x_n^*, \quad \hat{\pi}_f(p) \geq p \cdot y_f^* - c_f(y_f^*).$$

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Summing and rearranging,

$$\mathcal{S}^* = \sum_n u_n(x_n^*) - \sum_f c_f(y_f^*) \leq \sum_n \hat{u}_n(p) + \sum_f \hat{\pi}_f(p) - \underbrace{p \cdot \left(\sum_f y_f^* - \sum_n x_n^* \right)}_{\geq 0} \leq \sum_n \hat{u}_n(p) + \sum_f \hat{\pi}_f(p).$$

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Subtracting $\mathcal{S}(\mathbf{x}) = \sum_n u_n(x_n) - \sum_f c_f(y_f)$ obtains

$$\mathcal{S}^* - \mathcal{S}(\mathbf{x}) \leq \sum_n \underbrace{\hat{u}_n(p) - u_n(x_n) + p \cdot x_n}_{\mathcal{R}_n(x_n, p)} - p \cdot x_n + \sum_f \underbrace{\hat{\pi}_f(p) + c_f(y_f) - p \cdot y_f}_{\mathcal{R}_f(y_f, p)} + p \cdot y_f,$$

$$= \text{rationing losses} + \text{budget deficit.}$$

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(a) payments and allocations for sellers satisfy $y_f \in S_f(p)$ for all f,

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Applying **BFFWT** to $(p, \mathbf{x})$ and the envelope theorem, deadweight loss of a markup equilibrium is bounded by

$O(N\alpha)$ from N buyers' rationing losses $+$ $O(1)$ from counterfactual budget shortfall (overproduction).

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Thus $\alpha \leq O(1/N) \implies \mathbf{x}$ is $O(1)$-**approximately efficient**. This approximation is as good as we can hope for in environments where real shortfalls are unavoidable.

Key tool and contrast: Approximate equilibrium

Key tool: equilibrium approximation via the Shapley-Folkman Lemma.

Let $S_i \subseteq \mathbb{R}^L$ for $i = 1, ..., M$, and let $S = \bigoplus_{i=1}^{M} S_i$ be the Minkowski sum of those sets. Then any $x \in \text{co}(S)$ may be written as $x = \sum_{i=1}^{N} x_i$ where $x_i \in \text{co}(S_i)$ and $|i : x_i \in \text{co}(S_i) \setminus S_i| \leq L' := \min(L, M)$. Moreover, there exists $y, y' \in S$ such that $\|x - y\| \leq (\max_i r(S_i))\sqrt{L'}$ and $\|x - y'\| \leq (\max_i \rho(S_i))L'$.

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Applying to a convexified economy as in Starr (1969) and Heller (1972) gives **approximate equilibria**: price-allocation pairs $(p, \mathbf{x})$ with $\mathbf{x} \in \text{co}(\mathbf{X})$ such that:

- at most L agents have $x_n \in \text{co}(D_n(p)) \setminus D_n(p)$ or $y_f \in \text{co}(S_f(p)) \setminus S_f(p)$ (**rationing**), *or*
- there exists a nearby $\mathbf{x}' \in \mathbf{X}$ with $\|\mathbf{x} - \mathbf{x}'\|$ at most $O(\sqrt{L})$ such that $x'_n \in D_n(p)$ and $y'_f \in S_f(p)$ (**infeasibility**).

Constructing a markup equilibrium with α of $O(1/|N_t|)$

Simple markup equilibrium: Conduct a line search over $\alpha \geq 0$. For each α:

(i) Replace all buyers' values with $v_n/(1+\alpha)$
(ii) Replace values by concave envelope and costs with their convex envelopes.
(iii) Add an operational demand $R = \min\{r_{\mathcal{E}}\sqrt{L}, \rho_{\mathcal{E}}L\}$ for each good.
(iv) Find a Walrasian equilibrium of that economy.
(v) Round to a feasible allocation using Shapley-Folkman (using operational demand to cover shortfalls).
(vi) Set producer prices p and consumer prices $(1+\alpha)p$.

Iterate over $\alpha \geq 0$ to approximate the smallest α^* avoiding **budget shortfall**.

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Iterate over $\alpha \geq 0$ to approximate the smallest α^* avoiding **budget shortfall**.

Theorem: Let $|N_t| \to \infty$ with $|F_t|/|N_t| \to \phi$, such that there are finite choke prices and bounded nonconvexities, and suppose that $\mathcal{S}_t^*$ is $\Omega(|N_t|)$. Then the procedure above identifies an α of $O(1/|N_t|)$, leading to a deadweight loss of $O(1)$.

Computational properties

Conditional on α, the simple markup mechanism requires solving a convex optimization program. A wide class of such programs can be efficiently solved (e.g., strongly convex and self-concordant objectives).

Contrast: Efficient allocation problem in many nonconvex economies is often NP-hard.

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In practice: Ahunbay, Bichler, Dobos, and Knörr (2024) simulated the markup mechanism for European electricity markets, comparing it to standard "uplift" mechanisms, finding:

- Markup computations are **"considerably faster"** than uplift computations,
- uplift computations lead to **"substantial budget shortfalls"** while the markup mechanism leads to no budget shortfall, and
- markup allocations suffer only a **"small loss compared to the full optimum."**

Markup mechanisms also have better reporting incentives than uplift mechanisms.

Incentives

Markup mechanisms

Suppose now that u_n and c_f are **private information**, drawn from type spaces $\mathcal{U}$ and $\mathcal{C}$ respectively.

Focus on an **IPV** model: with probability ϕ a seller is drawn from full-support probability distribution χ on $\mathcal{C}$, with probability $1 - \phi$ a buyer is drawn from full-support probability distribution μ on $\mathcal{U}$.

Write D_μ and S_χ for the **expected** demand and supply correspondences.

A **markup mechanism** takes reports of u_n and c_f and outputs a markup equilibrium given those reports (with a mechanism to resolve multiplicity).

Interim incentives

A mechanism is ε-**interim IC** if, under truthful reporting, each player's payoff from *any report* is at most ε greater than the expected payoff of their truthful report.

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Theorem: If $\mathcal{U}$ and $\mathcal{C}$ are finite, *any* markup mechanism is $O(1/|N_t|^{\frac{1}{2}-\eta})-$interim IC for any $\eta > 0$.

Proof: Markup mechanisms are **envy-free**, thus **strategy-proof-in-the-large** (Azevedo & Budish, 2019). □

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Key assumption: Strongly monotone demand and supply (Watt, 2025).

Recall the law of demand: $(d(p) - d(p')) \cdot (p' - p) \geq 0$.

Strong monotonicity requires some m for which $(d(p) - d(p')) \cdot (p' - p) \geq m\|p - p'\|^2$ for all prices p, p' where $d(p)$ and $d(p')$ are not both $\{0\}$. With one good, equivalent to upper bound on demand's slope.

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Theorem: If D_μ and S_χ are strongly monotone, with probability $1 - O(1/|N_t|)$ over draws of (N_t, F_t), the simple and minimal markup mechanisms are $O(1/|N_t|^{1-\eta})$-ex post incentive compatible for any $\eta > 0$.

Proof: Relies on analogous results for Walrasian mechanisms established in (Watt,2025), using the fact that the simple markup equilibrium is a Walrasian equilibrium for an adjusted economy.

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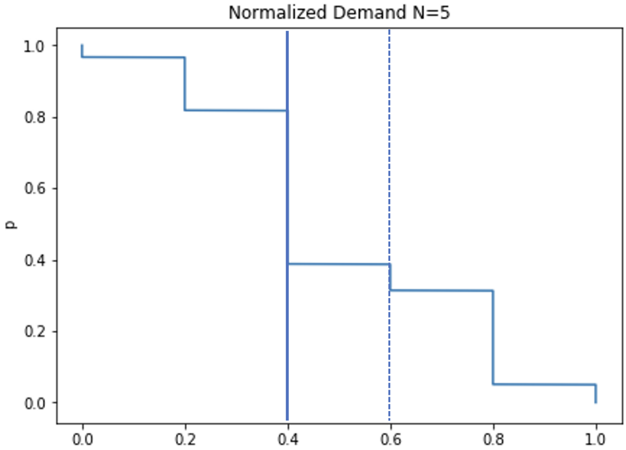
Corollary: The simple markup mechanism is $O(1/|N_t|^{1-\eta})$-IIC under the same assumptions.

Strong monotonicity intuition

Suppose N buyers have demand for *exactly* one unit with value $v \sim \mathrm{Unif}[0, 1]$, and there are $0.4N$ producers who can each produce a unit of the good at cost 0.



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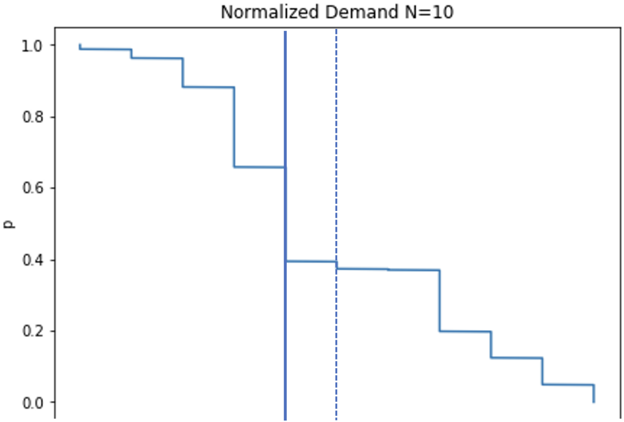

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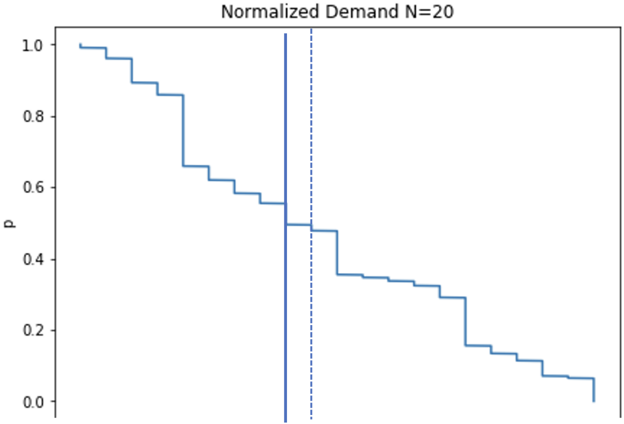

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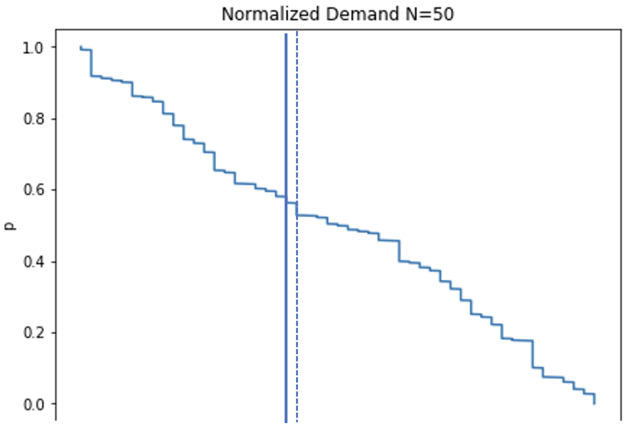

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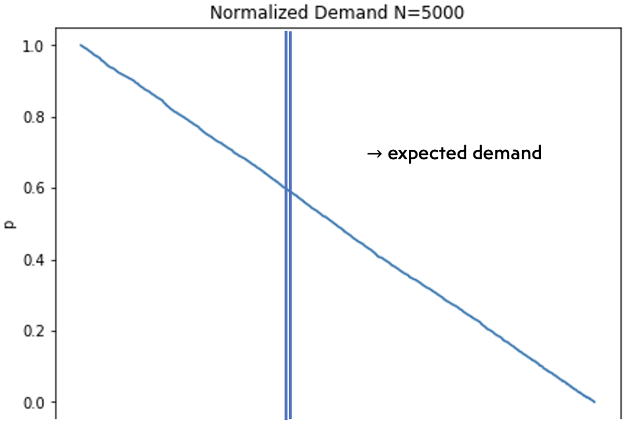

Each step (= single buyer's influence) becomes relatively small

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  - Transparent and practical market design.
- **Alternative approach:** rationing (after morning tea!)

Thank you for the invitation!