

Paying With Promises

Dynamic Contracting with Ex Post Participation

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Deakin Economic Theory Workshop

Motivation

Many relationships between a principal and an agent are governed by incomplete contracts.

This paper focuses on contracts with **ex post uncertainty** and **fixed payment terms**.

- ▶ E.g., legislated pay scales for teachers, master supply agreements, and trucking “rate cards” set payment terms before individual assignments are known.

Principals can condition assignments on the history of past allocations: **the promise of good jobs in the future can induce agents to take bad jobs today**.

- ▶ Remote service gives Queensland teachers priority for preferred locations; the U.S. Navy pairs sea tours with shore duty.

This Paper

I study a principal who assigns projects to agents under **fixed payment terms**.

- ▶ Each period, up to K projects can be assigned among $N \geq K$ agents.
- ▶ Project costs are realised and publicly observed.
- ▶ A task's **net flow value** for the agent in that period may be **positive or negative**.
- ▶ The principal benefits from completed projects and can commit to future assignments.
- ▶ An agent sees her assignment and promised future treatment, then may accept, refuse, or quit.

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Questions.

1. How should the principal use future assignments to induce agents to accept costly projects?
2. How does the relationship evolve in the long run?

Related Work

- ▶ **Payment terms and future business in practice.** Harris and Nguyen (2025); Blouin and Macchiavello (2019); Macchiavello and Morjaria (2015); Bernstein (2015); Chen et al. (2024).
- ▶ **Repeated projects and relational contracts.** Forand and Zápal (2020); Samuelson and Stacchetti (2017); Andrews and Barron (2016).
- ▶ **Assignment as an incentive.** Castro et al. (2021); Ji and Vong (2026); Eliaz et al. (2025).
- ▶ **Future allocation rights.** Bird and Frug (2019); Guo and Hörner (2018); Combe et al. (2025).
- ▶ **Recursive contracts.** Spear and Srivastava (1987); Abreu et al. (1990).

This paper: commitment to general, history-dependent assignment mechanisms; ex post participation after the realised assignment; potentially different discount factors for the principal and agents.

▶ Paper comparisons

Roadmap

1. Model
2. Single-agent benchmark
3. Optimal assignment rule
4. Long-Run Dynamics
5. Multiple agents and projects
6. Applications

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After observing the task and promised continuation, the worker accepts or rejects the assignment.

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3. If assigned, the worker observes the promised continuation and **chooses whether to accept**. Otherwise, the worker receives an outside option of 0 in that period.
4. Accepted tasks are completed; rejected or unassigned tasks disappear.

Payoffs

Let $x_t \in \{0, 1\}$ denote assignment and $y_t \in \{0, 1\}$ denote acceptance.

- ▶ Completion is $x_t y_t$.
- ▶ The firm receives **flow payoff** $x_t y_t$.
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- ▶ The firm receives **flow payoff** $x_t y_t$.
- ▶ The worker receives **flow payoff** $v_t x_t y_t$.
- ▶ **Discount factors** may differ: δ_P for the firm and δ_A for the worker.

The firm maximises $\mathbb{E} \left[(1 - \delta_P) \sum_{t \geq 0} \delta_P^t x_t y_t \right]$, while the worker evaluates $\mathbb{E} \left[(1 - \delta_A) \sum_{t \geq 0} \delta_A^t v_t x_t y_t \right]$.

Contracts

A contract is a **history-dependent** assignment rule.

At history h_t , the firm chooses **assignment probability** $X_t(v_t, h_t)$.

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I impose:

- ▶ **public histories**: task values, assignments, and acceptances are observed;
- ▶ **commitment**: the firm can commit to future assignment rules;
- ▶ **tie-breaking**: an indifferent worker accepts.

The constraint of interest is ex post participation after each task realisation.

Recursive Formulation

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- ▶ assignment $X(v; u) \in \{0, 1\}$;
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Notes:

- (a) It is without loss to restrict to **deterministic** contracts.
- (b) An optimal rule can be represented with every assigned task accepted; an **ex post participation constraint** makes acceptance optimal.
- (c) **Off-path**, the principal punishes the agent with the lowest feasible continuation promise (0).

Bellman Problem

The firm solves:

$$\Phi(u) = \max_{\substack{X(\cdot; u): \mathcal{V} \rightarrow \{0,1\} \\ z(\cdot; u): \mathcal{V} \rightarrow \mathcal{U}}} \mathbb{E}[(1 - \delta_p)X(v; u) + \delta_p \Phi(z(v; u))] ,$$

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subject to a **promise-keeping constraint**:

$$\mathbb{E}[(1 - \delta_A)vX(v; u) + \delta_A z(v; u)] \geq u \quad (\text{PK} - \lambda(u))$$

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and an **ex post participation constraint**:

$$z(v; u) \geq \underline{u}(v)X(v; u) \quad \text{for every } v, \quad \underline{u}(v) = \left[-\frac{1 - \delta_A}{\delta_A} v \right]_+ \quad (\text{PC} - \mu(v; u))$$

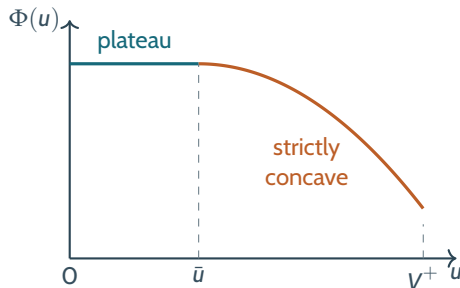
Value Function Properties

Proposition 1

The Bellman problem has a unique fixed point $\Phi : \mathcal{U} \rightarrow [0, 1]$. It is nonincreasing, concave, and continuous.

- ▶ There is a **plateau** $[0, \bar{u}]$: Φ is constant there.
- ▶ Above $\bar{u}$, promises are costly: Φ is **strictly decreasing** and **strictly concave** on $(\bar{u}, V^+]$.
- ▶ Above the plateau, the marginal cost of a larger promise is positive and rises with u .

Intuition for plateau: at $u = 0$, firm wants to assign positive-value tasks $\rightsquigarrow$ strictly positive utility (any negative value tasks assigned must be compensated by promises and contribute ≥ 0 to promised utility).



▶ Strict concavity

Optimal Single-Agent Contract

Static Benchmark

A **static** contract uses the same $X : \mathcal{V} \rightarrow \{0, 1\}$ after every history. Since each completed task gives the firm payoff 1, optimal static assignments are **cutoffs**: $X(v) = 1\{v \geq c\}$.

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For cutoff $c < 0$, the worker's payoff from accepting the task at the cutoff is

$$H(c) = \underbrace{\delta_A \mathbb{E}[v 1\{v \geq c\}]}_{\text{discounted value of future assignments}} + \underbrace{(1 - \delta_A)c}_{\text{current payoff at the cutoff}}.$$

Static cutoff c is **feasible** when $H(c) \geq 0$: future assignments compensate for taking the cutoff task.

Proposition 2: Optimal Static Contract

The **optimal static contract** is a cutoff rule $X^S(v) = 1\{v \geq c^S\}$, where

$$c^S = \begin{cases} \underline{v}, & \text{if } H(\underline{v}) \geq 0, \\ \text{the unique root of } H(c) = 0 \text{ in } (\underline{v}, 0), & \text{if } H(\underline{v}) < 0. \end{cases}$$

It delivers $U^S = \mathbb{E}[v 1\{v \geq c^S\}]$ and $\Pi^S = \Pr(v \geq c^S)$.

When is the static contract dynamically optimal?

When $\mathbb{E}[v] \geq \frac{1-\delta_A}{\delta_A}(-\underline{v})$, the firm can assign **every task** with the fixed promise $\mathbb{E}[v] \rightsquigarrow$ optimal.

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Otherwise, dynamic contracts do **strictly better**:

Proposition 4: Static Full Assignment or Dynamic Improvement

Either:

1. the optimal static contract has cutoff $c^S = \underline{v}$, so it assigns every task; or
2. the optimal static contract has cutoff $c^S > \underline{v}$, and the optimal dynamic contract strictly improves on the optimal static contract: $\Phi(0) > \Pi^S$.

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- ▶ A task just below the static cutoff needs only a little extra compensation, but completing it gives the principal a whole extra task.

Henceforth, assume $\mathbb{E}[v] < \frac{1-\delta_A}{\delta_A}(-\underline{v})$, so full assignment is statically infeasible.

▶ Proof sketch

Characterising the Optimal Dynamic Contract

At state u , the firm chooses $X : \mathcal{V} \rightarrow \{0, 1\}$ and $z : \mathcal{V} \rightarrow \mathcal{U}$ to solve

$$\Phi(u) = \max_{X, z} \mathbb{E}[(1 - \delta_P)X(v) + \delta_P \Phi(z(v))],$$

subject to

$$(\text{PK} - \lambda(u)) \quad \mathbb{E}[(1 - \delta_A)vX(v) + \delta_A z(v)] \geq u, \quad (\text{PC} - \mu(v; u)) \quad z(v) \geq X(v)\underline{u}(v) \quad \forall v.$$

1. **Solve for z given X .** Fix a candidate assignment rule and solve the continuation-promise problem.
2. **Given z , show optimal X is cutoff.** After substituting optimal continuation promises, show assignment is monotone in v .
3. **Solve jointly for X and z .** Combine the two steps to characterise the optimal contract.

The Optimal Dynamic Contract

The optimal policy has the following form for some cutoff $\gamma(u)$ (equivalently, target $z^T(u)$):

Assignment:

Continuation ($\bar{u} < u < V^+$):

$$X(v; u) = \mathbf{1}\{v \geq \gamma(u)\}.$$

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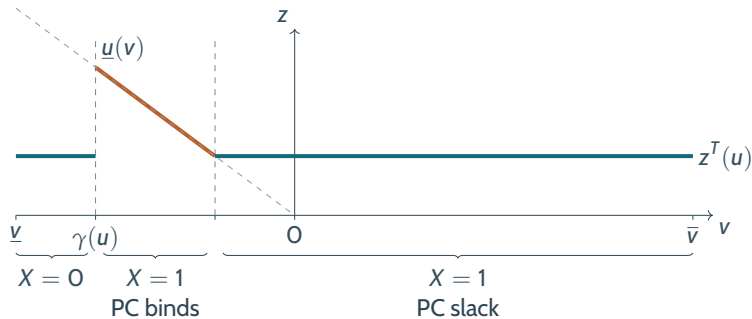
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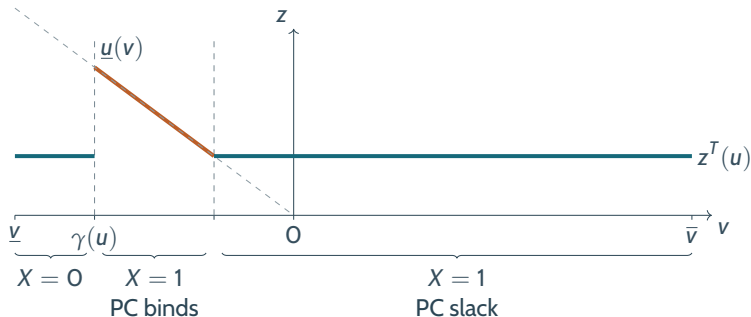
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What determines the cutoff $\gamma(u)$, or equivalently the slack target $z^T(u)$?

► Details

Characterising the Slack Target

Let $\lambda \geq 0$ and $\mu(v) \geq 0$ be the promise-keeping and participation multipliers. The Lagrangian is

$$\begin{aligned}\mathcal{L} = & \mathbb{E}[(1 - \delta_P)X(v) + \delta_P\Phi(z(v))] \\ & + \lambda \{ \mathbb{E}[(1 - \delta_A)vX(v) + \delta_A z(v)] - u \} \\ & + \mathbb{E}[\mu(v) \{ z(v) - \underline{u}(v)X(v) \}].\end{aligned}$$

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Along the optimal path, $u_{t+1} = z(v_t; u_t)$ and $\lambda_t \in \Lambda(u_t)$. The FOC gives the **Euler equation**:

$$\lambda_{t+1} = \underbrace{\frac{\delta_A}{\delta_p}}_q \lambda_t + \frac{\mu_t}{\delta_p} \in \Lambda(u_{t+1}).$$

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When participation is slack, $\mu_t = 0$ and $u_{t+1} = z^T(u_t)$, so $q\lambda_t \in \Lambda(z^T(u_t))$.

► Derivation

Long-Run Dynamics

Long-Run Dynamics: Three Regimes

Relative patience $q = \delta_A / \delta_P$ determines how promises and assignments evolve through the Euler equation:

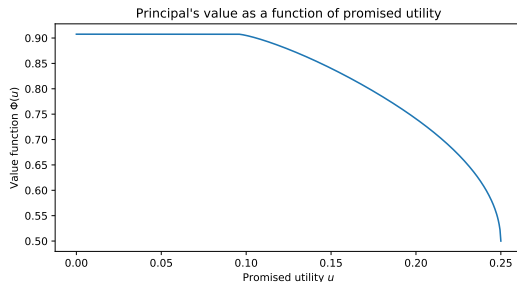
$$\lambda_{t+1} = q\lambda_t + \frac{\mu_t}{\delta_P} \in \Lambda(u_{t+1}), \quad \mu_t \geq 0.$$

Relative patience	Promised utility	Assignments
Agent more patient $q > 1$	Approaches the feasible maximum V^+ .	The cutoff approaches zero; negative-value tasks become increasingly rare.
Equal patience $q = 1$	Converges to an interior level determined by the history.	The cutoff settles at a negative level; mildly negative-value tasks remain eligible.
Principal more patient $q < 1$	Keeps fluctuating: falls when participation is slack and rises after difficult tasks.	Positive- and negative-value tasks continue to be assigned.

Agent More Patient: $q > 1$

Proposition 5(i) - Patient Agent

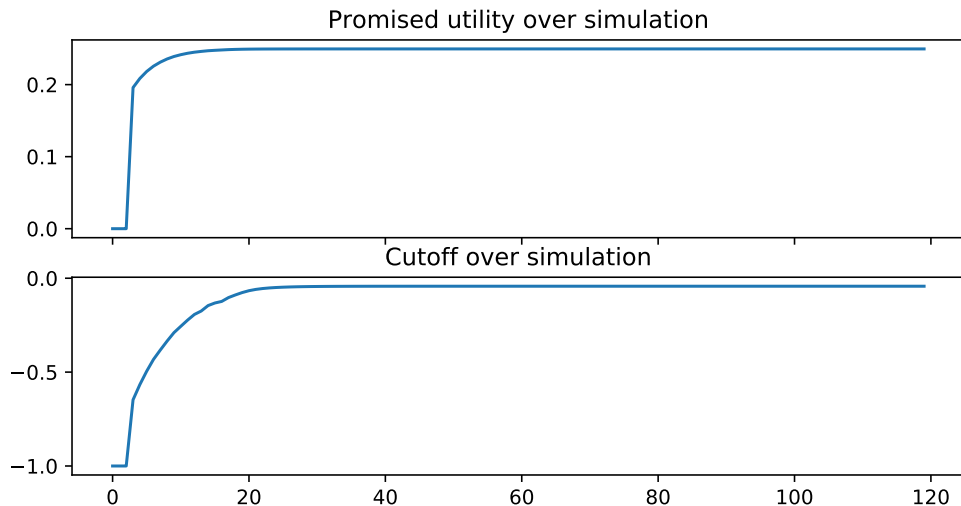
If $q > 1$, promised utility eventually rises toward its feasible maximum V^+ . The assignment cutoff approaches zero, making negative-value tasks increasingly rare.



- **Exchange rate:** $q > 1$ makes future compensation relatively cheap, favouring backloading.
- In contrast, cash payments can be deferred indefinitely (Sun and Tian, 2018).
- **Here, promises are fulfilled progressively through good jobs**, because feasible promises are bounded:
 $u_t \leq V^+ = \mathbb{E}[[v]_+]$.

Example Time Paths: $q > 1$

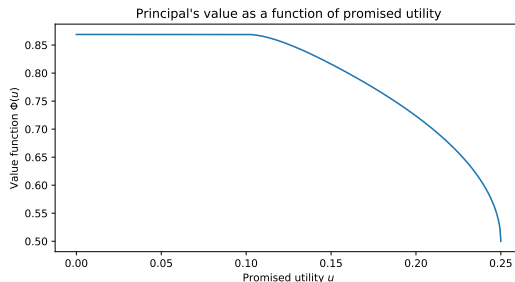
$$v \sim \text{Unif}[-1, 1], \delta_A = 0.8 > \delta_P = 0.7$$



Equal Patience: $q = 1$

Proposition 5(ii) - Equal Patience

If $q = 1$, promised utility converges to an interior level determined by the history. The cutoff approaches a fixed negative threshold, so mildly negative-value tasks remain eligible for assignment.

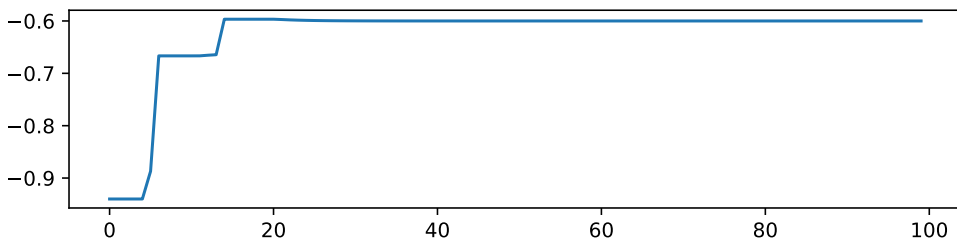
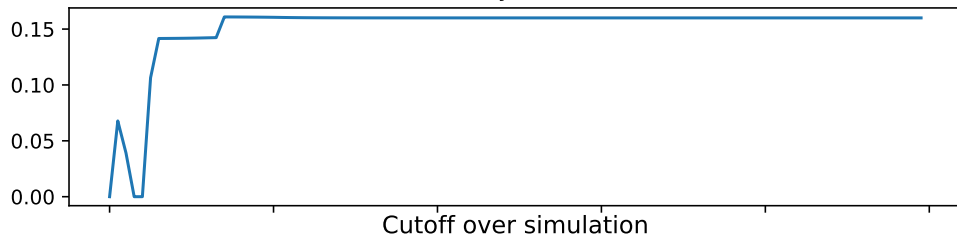


- **Equal discounting.** Both parties value the timing of compensation at the same rate. When participation is slack, $\lambda_{t+1} = \lambda_t$: the marginal cost of promises is equalised across dates.
- Difficult assignments can require additional promises, raising the costate when participation binds.
- Promises converge to an interior level determined by the history. The limiting cutoff remains negative, so mildly bad jobs continue to be assigned.

Example Time Paths: $q = 1$

$$v \sim \text{Unif}[-1, 1], \delta_A = \delta_P = 0.8$$

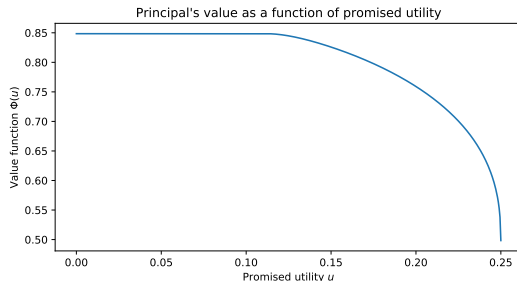
Promised utility over simulation



Principal More Patient: $q < 1$

Proposition 5(iii) - Patient Principal

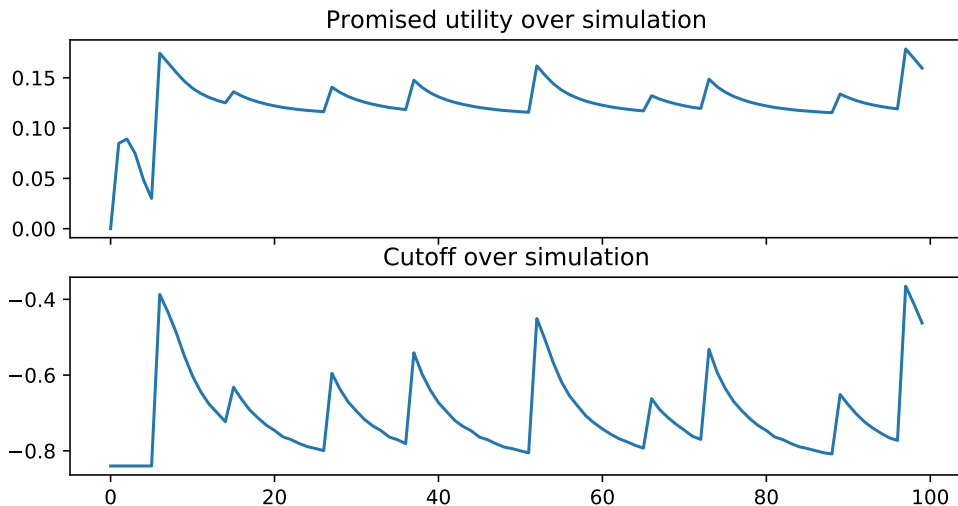
If $q < 1$, promised utility leaves the plateau in finite time and then keeps fluctuating. It falls toward the plateau when participation is slack and rises after some difficult assignments.



- ▶ **Future compensation is relatively expensive.** The agent discounts deferred rewards more heavily, so the principal fulfils promises through current assignments and runs down future obligations.
- ▶ With participation slack, $\lambda_{t+1} = q\lambda_t < \lambda_t$, moving the target promise $z^T(u_t)$ toward the plateau.
- ▶ Difficult assignments require promises of future good jobs, raising promised utility again.
- ▶ Positive- and negative-value assignments recur as promises are fulfilled and renewed, sustaining fluctuations in the relationship.

Example Time Paths: $q < 1$

$$v \sim \text{Unif}[-1, 1], \delta_A = 0.8 < \delta_P = 0.9$$



Optimal Multi-Agent Contract

From One Worker to Many

The only decision in the single-agent case is whether or not to assign the realised task. The multi-agent case adds the decision of who receives each assigned task.

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Objects

- ▶ $N \geq 2$ workers and $K \leq N$ tasks arrive each period.
- ▶ Workers share the same discount factor δ_A .
- ▶ Task values are $v = (v_1, \dots, v_K) \sim F^K$.
- ▶ The state is a promise vector $u = (u_1, \dots, u_N)$.

Choices and Constraints

- ▶ Assignment is a matching matrix $X(v; u) \in \{0, 1\}^{N \times K}$.
- ▶ Each task goes to at most one worker; each worker gets at most one task.
- ▶ Promise keeping and ex post participation hold worker by worker.

Feasible Promise Polytope

m workers can receive at most the best $\min\{m, K\}$ positive tasks per period. This bounds their total promise.

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$$\bar{V}_m = \mathbb{E} \left[\sum_{r=K-m+1}^K [v_{(r)}]_+ \right], \quad \bar{V}_0 = 0.$$

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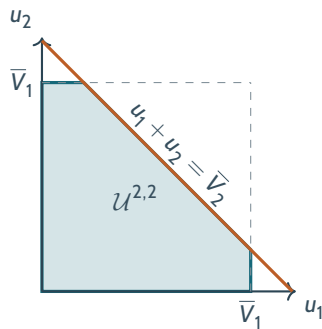
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A symmetric, downward-closed polytope.



Schur-Concavity and Equalisation

$\Phi^{N,K}$: continuous, nonincreasing, concave; a plateau. Worker **symmetry** adds a preference for equalisation.

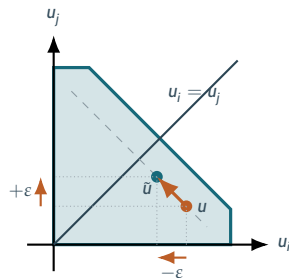
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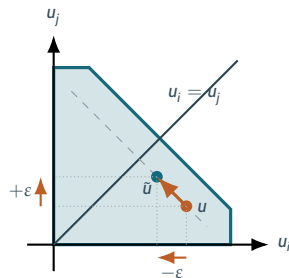
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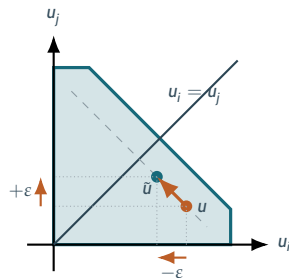
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⇒ Same total promise: more **equal** is weakly better for the principal.



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Optimal Assignment: Theorem and Intuition

Let $Y_i = \sum_k v_k X_{ik}$ be worker i 's current task value (0 if unassigned).

Theorem 2: Assortative Allocation on Average

There is an optimal rule with

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All positive tasks assigned; negative tasks assigned least costly first.

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Thus current utility enters as $(1 - \delta_A)\mathbb{E}[\sum_i \lambda_i Y_i]$. **Higher-costate workers receive more weight:** utility delivered today saves more costly future compensation.

Why Priority is Sometimes Soft

Distinct costates: sort tasks by marginal promise cost. If $\lambda_j > \lambda_i$ and $y > x$, giving y to j rather than i raises the current-utility term by

$$(1 - \delta_A)(\lambda_j - \lambda_i)(y - x) > 0.$$

The firm's completion payoff is unchanged. Participation may require an equalising continuation transfer; its costate-weighted loss cannot exceed this gain, and continuation value weakly rises.

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- ▶ $\lambda = 0$ supports plateau states; promises that are close to equal can also lead to tied costates $\lambda_i = \lambda_j$.
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Always giving j the better task may create too large a current gap. **Occasional reversals** keep assignments consistent with the promised gap; priority holds **on average**.

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- ▶ **Equalisation has economic value.** With symmetric workers, a more even distribution of a fixed total promise weakly improves the principal's payoff. Attractive tasks help fulfil larger promises; difficult tasks create claims for workers owed less.
- ▶ **The principal maximises its own payoff.** Its objective values completed tasks. Worker symmetry and the cost of fulfilling promises generate the force toward equalisation.

Long-Run Dynamics

Theorem 3: Long-Run Assignments

When a static rule cannot assign every task, relative patience determines which tasks continue to be assigned:

- ▶ **Agent more patient ($q > 1$):** negative-value tasks are assigned only finitely often.
- ▶ **Equal patience ($q = 1$):** positive- and negative-value tasks recur. Matching choices approach a stationary benchmark selected by the history.
- ▶ **Principal more patient ($q < 1$):** positive- and negative-value tasks recur. For an optimal rule, the worker taking the worst task regularly moves from the lowest to the highest priority group in the next period.

Takeaway. Future assignments compensate difficult tasks in the long run when the principal is at least as patient as the workers. With a more patient principal, priority reversals recur.

The Value of Flexible Assignment

Compare K separate relationships, each tied to one project stream, with a common pool of K agents and K projects per period. All relationships start with zero promises.

Pooling creates value

With at least two project streams, pooling strictly raises the principal's payoff whenever a static rule cannot assign every project.

- ▶ An attractive project in one stream can compensate an agent for accepting a difficult project in another.
- ▶ Flexibility lets the principal direct attractive projects toward larger claims and difficult projects toward smaller claims.
- ▶ **Procurement interpretation:** pool orders across products, plants, or markets to support cooperation through future business.

The Value of Spare Capacity

Can maintaining more agents than simultaneous projects improve incentives?

- ▶ **One project per period:** extra agents create no gross value. Only the total outstanding promise matters.
- ▶ **Several projects per period:** an extra agent can strictly improve the principal's payoff. The paper establishes examples of this gain.
- ▶ **Mechanism:** spare capacity allows an agent who accepted a very difficult project to receive favourable treatment while other agents handle the remaining work.
- ▶ **Applications:** spare staffing, an additional approved supplier, or a larger pool of platform providers.

Comparative Statics: Agent Patience and Project Values

Starting from zero promises, these results apply to both one and several agents.

- ▶ **More patient agents raise the principal's payoff.** The gain is strict whenever the principal falls short of assigning all projects. Future business becomes more effective compensation for difficult work.
- ▶ **The limits are intuitive.** As $\delta_A \rightarrow 0$, only positive-value projects are assigned in the limit. As $\delta_A \rightarrow 1$, the principal's discounted assignment fraction approaches one.
- ▶ **Better project values weakly raise the principal's payoff.** If agent values improve in the sense of first-order stochastic dominance, the old assignment process remains feasible.

Comparative Statics: Principal Patience

The principal's normalised payoff P measures the discounted number of assignments per period.

- ▶ **Greater principal patience weakly lowers P .** The decline is strict when static full assignment is infeasible. More weight falls on later periods, when earlier promises must be fulfilled.
- ▶ **Total discounted assignments increase.** The unnormalised sum, $P/(1 - \delta_P)$, strictly rises with principal patience.
- ▶ **The gain over the static benchmark shrinks.** A static rule delivers the same expected assignments each period; its normalised payoff is independent of principal patience.

Connection to long-run dynamics. When static full assignment is infeasible, difficult assignments recur once the principal is at least as patient as the agents.

Discussion

Motivating Example

Allocation of School Teachers



- ▶ Teachers are allocated centrally.
- ▶ Jobs appear over time: teacher retirements, transfers, and new demand.
- ▶ Assignment values vary with locational preferences and school difficulty.
- ▶ Public-sector pay scales limit salary adjustment across postings.
- ▶ **Transfer points:** teachers accrue priority while matched to less desirable schools.

▶ [More details](#)

Queensland Teacher Transfers

	Year 1	Year 2	Year 3	Year 4	Year 5
Rating 7	11	11	17	20	22
Rating 6	7	7	7	10	12
Rating 5	5	5	5	7	8
Rating 4	4	4	4	6	7
Rating 3	3	3	3	4	5
Rating 2	2	2	2	2	2
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Transfer points by school rating and years of service.

Institution.

- ▶ Teachers are allocated through a centralised transfer system.
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Interpretation. Hardship service builds standing for future transfers.

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- ▶ **Kenyan roses.** Supply disruptions force exporters to prioritise orders; deliveries depend on the value of established relationships. [Macchiavello and Morjaria \(2015\)](#).

Other Assignment Currencies

Queue priority

- ▶ Uber, Lyft, and DiDi use airport or short-trip queue rules.
- ▶ A bad trip preserves or upgrades future access to better trips.

Point systems

- ▶ Amazon Flex drivers accumulate standing through completed blocks.
- ▶ Higher standing unlocks preferred stations, times, and blocks.

Rotation rules

- ▶ Foreign service and naval careers link hardship tours to later desirable postings.
- ▶ Residency programmes use point bidding for preferred rotations.

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Further analysis: optimal turnover requires hiring and termination costs, and a rule for settling promises through severance, buyouts, or future business.

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- ▶ **The continuation logic carries over.** Under suitable conditions, the single-agent promise is the larger of a target and the minimum required to induce the recommended action.

Conclusion

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- ▶ With payment terms fixed in advance, future projects can compensate agents who accept costly assignments today.
- ▶ With one agent, the principal assigns projects above a cutoff that rises weakly with the agent's promised utility.
- ▶ Relative patience determines whether negative-value projects gradually disappear or keep recurring.
- ▶ With several agents, an optimal rule gives agents owed more assignments worth at least as much on average. Similar promises can allow reversals in individual periods.
- ▶ **Applications:** Transfer points, preferred-supplier status, and queue priority reward current service with access to future opportunities.

Less Obvious Implications

- ▶ **Patient workers need not do difficult work forever.** When static full assignment is infeasible, negative-value assignments eventually stop if agents are more patient than the principal. They recur with positive long-run frequency if the principal is at least as patient.

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- ▶ **Priority need not be a strict queue.** An optimal rule gives workers with higher promises assignments worth at least as much on average. Tied costates can allow reversals in individual periods.

Appendix

Solving for z given X

Fix $u \in (\bar{u}, V^+)$ and an assignment rule X . Apart from participation, v enters only through **average current utility** $\mathbb{E}[vX(v)]$:

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Choose a **common target** z_λ^T , unless participation requires more:

$$z_X(v; \lambda) = \max\{z_\lambda^T, \ell_X(v)\}, \quad \ell_X(v) = X(v; u)u(v).$$

Solving for z given X

Fix $u \in (\bar{u}, V^+)$ and an assignment rule X . Apart from participation, v enters only through **average current utility** $\mathbb{E}[vX(v)]$:

$$\underbrace{\delta_A \mathbb{E}[z(v)]}_{\text{future utility contribution}} \geq \underbrace{u}_{\text{utility owed}} - \underbrace{(1 - \delta_A) \mathbb{E}[vX(v)]}_{\text{current utility contribution}}.$$

At marginal promise cost λ , every state has the **same continuation objective**: $\delta_P \Phi(z) + \delta_A \lambda z$. It does not depend on realised v ; concavity favours smoothing promises across states.

Choose a **common target** z_λ^T , unless participation requires more:

$$z_X(v; \lambda) = \max\{z_\lambda^T, \ell_X(v)\}, \quad \ell_X(v) = X(v; u)u(v).$$

- ▶ $\ell_X(v) \leq z_\lambda^T$: the same promise $z = z_\lambda^T$ across these states.
- ▶ $\ell_X(v) > z_\lambda^T$: participation binds; promise just enough for acceptance.

Optimal Assignments are Cutoff Rules

Theorem 1(i): Assignment

There is a cutoff $\gamma : [0, V^+] \rightarrow [\underline{v}, 0]$ such that $X(v; u) = \mathbf{1}\{v \geq \gamma(u)\}$. The cutoff is constant on $[0, \bar{u}]$, nondecreasing above $\bar{u}$, and $\gamma(V^+) = 0$.

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Why a cutoff? Fix the marginal promise cost λ . Compared with a lower-value task, a higher v gives:

1. the **same payoff to the firm** from completion;
2. **more current utility to the worker**, helping satisfy promise keeping;
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Not assigning gives the same continuation target z_λ^T for every v . Thus the gain from assigning is **nondecreasing in v** : if a task is worth assigning, every higher-value task is too. This gives a cutoff.

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Why does the cutoff rise with u ? The cutoff balances the payoff from completion against the **marginal cost of additional promises**, captured by λ . Above the plateau, higher u raises λ , making future obligations more costly.

[◀ Back](#)

Deriving the Costate Equation

Fix state u_t , realised task value v_t , and choose $\lambda_t \in \Lambda(u_t)$. The terms in the Lagrangian that depend on the continuation promise $u_{t+1} = z(v_t; u_t)$ are

$$\delta_P \Phi(u_{t+1}) + \delta_A \lambda_t u_{t+1} + \mu_t(v_t) \{u_{t+1} - \underline{u}(v_t) X_t(v_t)\}.$$

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- ▶ Slack participation: $\mu_t(v_t) = 0$, so the costate moves by relative patience q .
- ▶ A positive multiplier requires binding participation and raises the next costate.

Strict Concavity: Intuition

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Promises are bounded and $\delta_A < 1$, so the largest gap is zero. Each positive marginal cost selects **one promise**, ruling out linear segments above the plateau.

[▶ Proof](#)[▶ Back](#)

Proof: Strict Concavity

Let $A_\lambda = \arg \max_{u \in [0, V^+]} \{\Phi(u) + \lambda u\}$: the promises supported by marginal cost $\lambda > 0$. Define the largest gap $M = \sup_{\lambda > 0} \text{diam}(A_\lambda) \leq V^+$.

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1. **Current assignments coincide.** For $u, \hat{u} \in A_\lambda$, the optimal cutoff is the same almost surely. Binding promise keeping gives $u - \hat{u} = \delta_A \mathbb{E}[z(v) - \hat{z}(v)]$.

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Every positive costate selects a single promise. A linear segment above the plateau would have the same positive costate at multiple promises, so Φ is **strictly concave** there.

Proof: Proposition 4

Static full assignment gives payoff 1 every period, so it is dynamically optimal whenever it is feasible.

Suppose instead $c^S > \underline{v}$. Then $c^S \in (\underline{v}, 0)$, static feasibility binds, and $U^S = \mathbb{E}[v \mathbf{1}\{v \geq c^S\}] = \underline{u}(c^S) < V^+$.

1. Pick small $\varepsilon > 0$ and lower the cutoff to $c_\varepsilon = c^S - \varepsilon$.
2. Assign all $v \geq c_\varepsilon$. For assigned negative values, set $z_\varepsilon(v) = \underline{u}(v)$; otherwise set $z_\varepsilon(v) = 0$.
3. Participation holds by construction, and promise keeping at $u = 0$ holds because positive tasks generate slack.
4. The new marginal interval has probability $m_\varepsilon = \Pr(c_\varepsilon \leq v < c^S) > 0$.
5. Continuity of Φ gives dynamic payoff at least $\Pi^S + (1 - \delta_p - \delta_p \eta)m_\varepsilon$, which exceeds Π^S for small η .

Thus, when static full assignment is infeasible, the dynamic contract strictly improves on the static optimum.

Proof: Concavity and Symmetry Imply Schur-Concavity

Let Φ be concave and symmetric on a convex symmetric promise set. Fix a Robin Hood transfer from i to j : $\tilde{u} = u - \varepsilon e_i + \varepsilon e_j$, with $u_i > u_j$ and $0 < \varepsilon \leq (u_i - u_j)/2$.

Let σu swap coordinates i and j , and set $\alpha = \varepsilon / (u_i - u_j) \in (0, 1/2]$. Then $\tilde{u} = (1 - \alpha)u + \alpha \sigma u$.

By concavity and symmetry, $\Phi(\tilde{u}) \geq (1 - \alpha)\Phi(u) + \alpha\Phi(\sigma u) = \Phi(u)$.

Thus every Robin Hood transfer weakly raises the value function. Repeating the argument along any finite sequence of transfers gives $u \succeq \tilde{u} \Rightarrow \Phi(\tilde{u}) \geq \Phi(u)$.

► Back

Related Papers

Related Papers

- ▶ Sun and Tian (2018)
- ▶ Samuelson and Stacchetti (2017)
- ▶ Combe, Nora and Tercieux (2025)
- ▶ Forand and Zápal (2020)
- ▶ Andrews and Barron (2016)
- ▶ Bird and Frug (2019)
- ▶ Ji and Vong (2026)

Sun and Tian (2018)

Model and Results

Optimal Contract to Induce Continued Effort, Management Science.

- ▶ **Model.** A risk-neutral agent privately chooses costly effort that raises the arrival rate of Poisson successes. The principal observes successes and earns revenue from them.
- ▶ **Instruments.** The principal commits to nonnegative cash transfers and possible termination. Promised utility summarises the history.
- ▶ **Equal patience.** Promised utility reaches either zero, ending the relationship, or an absorbing payment threshold, followed by constant success bonuses.
- ▶ **Principal more patient.** Promises decline between successes and jump after successes. The payment threshold is nonabsorbing; the baseline relationship terminates in finite time almost surely.

Scope: contracts inducing high effort until termination.

▶ Comparison

◀ Related papers

Sun and Tian (2018)

Similarities and Differences

- ▶ **Similarities.** Both use future compensation to provide incentives, and relative patience shapes the evolution of promises.
- ▶ They do not hold payment terms fixed. Cash bonuses cap continuation rewards at a payment threshold; here, a difficult job can require $z(v; u) > z^T(u)$.
- ▶ They do not have our ex post acceptance constraint for publicly observed task values. Their incentive problem is hidden effort.
- ▶ They do not derive the promise bound from future good jobs. Here, assignment values imply $u \leq V^+$.
- ▶ Their baseline $q < 1$ contract terminates almost surely. Here, promises keep fluctuating within the relationship when static full assignment is infeasible.

[◀ Model](#)[◀ Related papers](#)

Samuelson and Stacchetti (2017)

Model and Results

Even Up: Maintaining Relationships, Journal of Economic Theory.

- ▶ **Model.** Two players repeatedly observe an i.i.d. project that benefits one player and costs the other. Both must approve; the person bearing the cost varies across projects.
- ▶ **Self-enforcement.** The baseline has public projects and actions and a common discount factor. Future cooperation must make approval optimal for both players.
- ▶ **Future projects as compensation.** Without transfers, sufficiently costly accepted projects reward the person bearing the cost through better future treatment.
- ▶ **Voluntary transfers.** Efficient relationships use continuation play that maximises total surplus. Self-enforcing payments handle costly projects and can occur infrequently.

▶ Comparison

◀ Related papers

Samuelson and Stacchetti (2017)

Similarities and Differences

- ▶ **Similarities.** In their model without transfers, future good projects compensate for accepting costly projects, as they do here.
- ▶ They do not give one player commitment power. Both must prefer to honour the relationship; here, the principal commits to future assignments.
- ▶ They do not allow different discount factors. We study all three relative-patience regimes.
- ▶ They do not study assignment across several agents. Our principal can allocate scarce projects among several agents.
- ▶ They characterise the frontier of bilateral self-enforcing payoffs. We solve the principal's optimal assignment policy with commitment and ex post participation.

[◀ Model](#)[◀ Related papers](#)

Combe, Nora and Tercieux (2025)

Model and Results

Dynamic Assignment without Money: Optimality of Spot Mechanisms, Theoretical Economics.

- ▶ **Model.** A finite horizon and a continuum of agents; each agent must be assigned a perishable object at every date.
- ▶ **Private preferences.** Agents know their preferences over complete assignment sequences at date zero; preferences allow complementarities across dates.
- ▶ **Spot mechanism.** Each agent receives an initial random token budget, buys assignments at dated prices, and carries unused tokens forward.
- ▶ **Ordinal result.** Under full-support preferences, incentive compatibility and ordinal efficiency characterise spot mechanisms.
- ▶ **Cardinal result.** With cardinal reports and stronger regularity and support assumptions, incentive compatibility and Pareto efficiency characterise spot mechanisms with menus of initial budget lotteries.

[▶ Comparison](#)[◀ Related papers](#)

Combe, Nora and Tercieux (2025)

Similarities and Differences

- ▶ **Similarities.** Both use accounts to link assignments over time and give agents access to future opportunities.
- ▶ They do not impose our ex post participation constraint: their baseline requires an assignment each date. Our agent can refuse or quit after observing the task value.
- ▶ They do not have unexpected preference shocks. Agents know their preferences over complete assignment paths at date zero; here, public task values are realised each period.
- ▶ They characterise efficient, incentive-compatible mechanisms. We maximise the principal's payoff subject to promise keeping and ex post participation.
- ▶ They use a finite horizon and a continuum of agents. We use an infinite horizon and finitely many agents.

Section 7 permits a null assignment and planner optimisation within the efficient class.

[◀ Model](#)[◀ Related papers](#)

Forand and Zápal (2020)

Model and Results

Production Priorities in Dynamic Relationships, Theoretical Economics.

- ▶ **Model.** A principal commits to production decisions; an agent can quit. Public projects yield fixed utility pairs and follow an arbitrary stochastic process. Both parties share a discount factor.
- ▶ **Timing.** The agent observes the project and chooses whether to remain before a public lottery determines production.
- ▶ **Production priorities.** Demands favour high principal benefits relative to agent costs. Rewards favour high agent benefits relative to principal costs.
- ▶ **Dynamics.** Reward commitments broaden toward less cost-effective projects, while demands narrow. With finitely many project types, long-run inefficiency vanishes.

[▶ Comparison](#)[◀ Related papers](#)

Forand and Zápal (2020)

Similarities and Differences

- ▶ **Similarities.** A committed principal uses favourable future projects to compensate current sacrifice; stochastic opportunities determine the available rewards.
- ▶ They do not require participation after the production lottery is realised. Here, the agent can reject the realised assignment.
- ▶ They do not study different discount factors. We study all three relative-patience regimes.
- ▶ They do not require every completed project to benefit the principal. Here, good jobs benefit both parties and also deliver compensation.
- ▶ They do not study competing agents. Here, scarce projects link the promises made to several workers.

The participation-timing difference matters when production is randomised.

▶ Participation example

◀ Model

◀ Related papers

Forand and Zápal (2020)

Participation and the Production Lottery

A bad job costs the agent 1 unit of utility today. Promised future utility is worth $1/2$ today, whether or not the job is assigned.

- ▶ **Forand–Zápal.** After observing the job, the agent commits to a 50% chance of production. The assigned outcome gives $-\frac{1}{2}$; the unassigned outcome gives $+\frac{1}{2}$. They can **bundle good and bad outcomes** into an acceptable lottery with expected utility 0.
- ▶ **Here.** The agent can refuse the realised assignment. Every realised assignment must satisfy participation, so the agent rejects the bad outcome.
- ▶ **Our deterministic reduction.** It applies only to lotteries where **every realised assignment satisfies participation**. For a given deterministic policy, the participation constraints coincide.

[◀ Comparison](#)[◀ Related papers](#)

Andrews and Barron (2016)

Model and Results

The Allocation of Future Business: Dynamic Relational Contracts with Multiple Agents, AER.

- ▶ **Model.** A principal selects one supplier after public productivity draws. The supplier chooses hidden effort; output and payments are observed within that relationship.
- ▶ **Relational incentives.** Cash payments are flexible and promises must be self-enforcing. Future business makes promised incentive payments credible.
- ▶ **Favoured supplier.** Among the currently most productive suppliers, choose the one who succeeded most recently. Subsequent failures by that supplier do not remove its priority.
- ▶ **Result.** Under their parameter condition, this rule achieves efficient supplier selection and high effort whenever any relational contract can. It improves on stationary rules.

[▶ Comparison](#)[◀ Related papers](#)

Andrews and Barron (2016)

Similarities and Differences

- ▶ **Similarities.** Future business provides incentives, and scarce assignments link several ongoing relationships.
- ▶ They do not hold cash payments fixed. Here, compensation comes through future good jobs under fixed payment terms.
- ▶ Their principal cannot commit. Future business sustains the credibility of bonuses; our principal commits to future assignments.
- ▶ They do not make every relationship's history public. Their incentive problem combines hidden effort with private bilateral histories; our task values and histories are public.
- ▶ They do not allow different discount factors. We study all three relative-patience regimes and the resulting allocation dynamics.

[◀ Model](#)[◀ Related papers](#)

Bird and Frug (2019)

Model and Results

Dynamic Non-monetary Incentives, AEJ: Microeconomics.

- ▶ **Model.** A committed principal and one agent share a discount rate. Investment and reward opportunities arrive through separate Poisson processes.
- ▶ **Private opportunities.** The agent privately observes arrivals, while completed actions are public. Investments benefit the principal and cost the agent; rewards reverse these payoffs.
- ▶ **Identical rewards.** Optimal compensation gives the agent permission to take every reward arriving within a predetermined time window.
- ▶ **Several reward types.** The optimal mechanism uses separate time allowances for different rewards.

[▶ Comparison](#)[◀ Related papers](#)

Bird and Frug (2019)

Similarities and Differences

- ▶ **Similarities.** Stochastic opportunities limit compensation; future access to rewards induces costly current actions.
- ▶ Their baseline does not make opportunity arrivals public. Our task values are publicly observed, and acceptance follows each realised assignment.
- ▶ They do not use one task stream that always benefits the principal. Their investments and rewards have opposite payoff effects; here, good jobs benefit both parties.
- ▶ They do not study different discount rates. We study all three relative-patience regimes.
- ▶ They study one agent. Our principal can allocate several simultaneous projects among several agents.

Section IV.B also studies public investment opportunities and the agent's ability to refuse.

[◀ Model](#)[◀ Related papers](#)

Ji and Vong (2026)

Model and Results

Task Assignment with Interdependent Incentives, September 2026 working paper.

- ▶ **Model.** A manager assigns one burden among identical workers. Effort is private; success or failure is public.
- ▶ **Instruments.** Assignment provides incentives. The baseline has no cash, no managerial commitment, and a common discount factor.
- ▶ **Optimal priority.** Among equilibria inducing effort whenever assigned, retain the worker after failure and rotate after success. Vacancy phases strengthen incentives when necessary.
- ▶ **Reward and inequality.** Success earns relief until other workers succeed and the vacancy phase ends. Queue positions give identical workers different expected discounted future workloads.

▶ Comparison

◀ Related papers

Ji and Vong (2026)

Similarities and Differences

- ▶ **Similarities.** Future assignment provides incentives, and rewarding one worker changes the opportunities available to others.
- ▶ They induce hidden effort. We induce acceptance after publicly observed task values.
- ▶ Their baseline manager cannot commit. Our principal commits to future assignments.
- ▶ Their baseline has a burden task and common discounting. We allow good and bad jobs within the same relationship and different discount factors.
- ▶ They do not analyse multiple simultaneous tasks in this version. We allow up to K projects to be assigned among $N \geq K$ agents.

Section 6 adds participation constraints, desirable tasks, and other extensions.

[◀ Model](#)[◀ Related papers](#)